Homework 4:

I pledge my honor that I have abided by the Stevens honor system.

2.1: 10, 26, 30, 42; 2.2: 4, 8a, 12, 16abc, 20; 2.3: 10, 16, 20

2.1:

10. Determine whether these statements are true or false.

1. ∅ ∈ {∅} True
2. ∅ ∈ {∅, {∅}} True
3. {∅} ∈ {∅} False
4. {∅} ∈ {{∅}} True
5. {∅} ⊂ {∅, {∅}} True
6. {{∅}} ⊂ {∅, {∅}} False
7. {{∅}} ⊂ {{∅}, {∅}} True

26. Show that if A ⊆ C and B ⊆ D, then A x B ⊆ C x D.

1. A ⊆ C ≡ ∀x (x ∈ A → x ∈ C) definition of ⊆
2. B ⊆ D ≡ ∀y (y ∈ B → y ∈ D) definition of ⊆
3. (x, y) ∈ A x B relation from A to B
4. (x, y) ∈ C x D relation from C to D
5. A x B ⊆ C x D

30. Suppose that A x B = ∅, where A and B are sets. What can you conclude?

Either A, B, or both are empty sets (∅).

42. Translate each quantification into English and determine the truth-value.

1. ∃x ∈ **R** (x3 = −1): There exists a real number, x, such that x3 = −1.

True (x = −1: (−1)3 = −1)

1. ∃x ∈ **Z** (x + 1 > x): There exists an integer x such that x + 1 > x.

True (x = 1: 1 + 1 = 2 > 1)

1. ∀x ∈ **Z** (x − 1 ∈ **Z**): For all integers x, x − 1 is also an integer.

True: Domain is **Z**, so Range is **Z**.

1. ∀x ∈ **Z** (x2 ∈ **Z**): For all integers x, x2 is also an integer.

True: Domain is **Z** (−∞, ∞), Range is **Z** (0, ∞)

2.2:

4. Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}. Find:

1. A ∪ B: {x | x ∈ A ∨ x ∈ B} => {a, b, c, d, e, f, g, h}
2. A ∩ B: {x | x ∈ A ∧ x ∈ B} => {a, b, c, d, e}
3. A − B: {x | x ∈ A ∧ x ∉ B} => { }
4. B − A: {x | x ∈ B ∧ x ∉ A} => {f, g, h}

8. a. Prove A ∪ A = A.

∀x (x ∈ (A ∪ A) ↔ x ∈ A)

1. x ∈ A ∪ A definition of ∪
2. x ∈ {y| y ∈ A ∧ (y ∈ A ∨ y ∈ A)} set builder notation
3. x ∈ A ∨ x ∈ A idempotence of ∨
4. x ∈ A

12. Prove A ∪ (A ∩ B) = A.

∀x (x ∈ A ∨ (x ∈ A ∧ x ∈ B) ↔ x ∈ A)

1. x ∈ A ∪ (A ∩ B) definition of ∪, ∩
2. x ∈ {z | z ∈ A ∨ (z ∈ A ∧ z ∈ B)} set builder
3. x ∈ A ∨ (x ∈ A ∧ x ∈ B) x ∈ A in both cases, therefore.
4. A ∪ (A ∩ B) ⊆ A switch directions…
5. y ∈ {z | z ∈ A ∨ (z ∈ A ∧ z ∈ B)} set builder
6. y ∈ A ∨ y ∈ (A ∩ B) definition of ∪, ∩
7. y ∈ A ∪ (A ∩ B) y ∈ A in both cases
8. A ⊆ A ∪ (A ∩ B) definition of absorption law
9. A ∪ (A ∩ B) = A

16. Let A and B be sets. Show that:

1. (A ∩ B) ⊆ A

∀x (x ∈ (A ∩ B) → x ∈ A)

1. x ∈ (A ∩ B) definition of ∩
2. x ∈ {y | y ∈ A ∧ y ∈ B) set builder
3. x ∈ A ∧ x ∈ B A intersect B is a subset of A
4. (A ∩ B) ⊆ A
5. A ⊆ (A ∪ B)

∀x (x ∈ A → x ∈ (A ∪ B))

1. x ∈ A ∨ x ∈ B definition of ∨
2. x ∈ A ∪ B definition of ∪
3. x ∈ A → x ∈ A ∪ B if an element exists in a subset, then it exists in the set
4. x ∈ A → x ∈ A ∨ x ∈ B definition of ∨
5. A ⊆ (A ∪ B) x is in A or B, A is a subset or equal to (A ∪ B)
6. A − B ⊆ A

∀x ((x ∈ A ∧ x ∉ B) → x ∈ A)

1. x ∈ A − B definition of difference
2. x ∈ {y | y ∈ A ∧ y ∉ B} set builder
3. x ∈ A ∧ x ∉ B if x isn’t in B, then it’s just in A
4. x ∈ A the difference of A and B is a subset of A
5. A − B ⊆ A

20. Show that if A and B are sets with A ⊆ B, then:

* 1. A ∪ B = B

If A ⊆ B, then A ∈ B ∧ (x ∈ A ∨ x ∈ B).

Therefore, x ∈ B.

Therefore, A ∪ B = B

* 1. A ∩ B = A

If A ⊆ B, then A ∈ B ∧ (x ∈ A ∧ x ∈ B)

Therefore, A = B → A ∩ B = A

2.3:

10. Determine whether each one of these functions from {a, b, c, d} to itself is one-to-one.

* 1. f(a) = b, f(b) = a, f(c) = c, f(d) = d True
  2. f(a) = b, f(b) = b, f(c) = d, f(d) = c False
  3. f(a) = d, f(b) = b, f(c) = c, f(d) = d False

16. Consider these functions from the set of students in a discrete mathematics class. Under what conditions is the function one-to-one if it assigns to a student his or her:

* + 1. Mobile phone number: every phone has its own phone number.
    2. Student id number: the school gives each student a unique id number.
    3. Final grade in the class: no two students get the same grade in the class.
    4. Hometown: no two students are from the same town.

20. Give an example of a function form **N** to **N** that is:

1. One-to-one but not onto: f(x) = x + 1
2. Onto but not one-to-one: f(x) = x/2
3. Both onto and one-to-one (different from identity):

f(x) =

1. Neither one-to-one nor onto: f(x) = c where c is a constant.